Physics 200A **Theoretical Mechanics** **Fall 2013**

Problem Set III: Due Wednesday, October 23, 2013
Discussed in Problem Session on Tuesday, October 22, 2013

1.) A uniform bar of mass M and length 2ℓ is suspended from one end by a spring of force constant k. The bar can swing freely in one vertical plane. The spring moves only vertically. Derive the Hamiltonian and the Hamiltonian equations of motion.

2.) Fetter and Walecka (FW): 6.1

3.) FW: 6.2

4.) FW: 6.4

5.) Derive Hamilton's equations directly from a modified version of the Principle of Least Action.

6.) Consider the Helmholtz equation for a sound wave in a medium with index of refraction $n(\underline{x})$.

$$\nabla^2 \psi + \frac{\omega^2}{c_0^2} n(\underline{x})^2 \psi = 0 .$$

a.) For $n(\underline{x})^2 = 1 + \delta(\underline{x})$, where $\delta << 1$, and assuming sound is beamed in the $\hat{z}$ direction, show the Helmholtz equation may be (approximately) re-written as:

$$2i\, k_z \frac{\partial \psi}{\partial z} + \nabla_\perp^2 \psi + \frac{\omega^2}{c_0^2} \delta(\underline{x}) \psi = 0 .$$

b.) Define k_r here. The above equation is called the parabolic wave equation. Discuss

i.) the approximations inherent to this formulation.

ii.) the physical meaning of the different terms.

iii.) the restrictions on $\partial \psi / \partial z$, etc.

c.) Now, write $\psi = A(\underline{x})e^{i\phi(\underline{x})}$. Use the parabolic wave equation to derive coupled equations for phase $\phi(\underline{x})$ and amplitude $A(\underline{x})$. Discuss the physical content of your result. Can you relate your result to that obtained using eikonal theory?

d.) Extra Credit:

What happens if $\delta_{(x)}$ is stochastic, so $\left\langle \delta_{(x)}\delta_{(x')} \right\rangle = \delta_0^2 c(x-x')$. How would you calculate ψ ?

7.) Consider an ocean with sound speed a function of depth, so that $c_s(z)$ is maximal at z_0, where z is depth as measured from the surface. Using Fermat's Principle, determine the path that a ray takes to traverse a long distance ℓ.